1. Introduction

On 8 October 2013, near-Earth object (NEO) 2013 TV_{135} was discovered at the Crimean Astrophysical Observatory in Ukraine^{*}. Heliocentric motion of 2013 TV_{135} from August 2013 through nearly one orbit period to May 2017 is plotted in Figure 1[†]. Note how this NEO was closest to Earth in mid-September, weeks before its discovery. Because closest approach was inside Earth's orbit, however, observing 2013 TV_{135} was difficult at that time. From 1 September until 10 October 2013, 2013 TV_{135} was less than 100° from the Sun in Earth's sky, but minimum solar elongation was an observable 77.3° on 18 September.

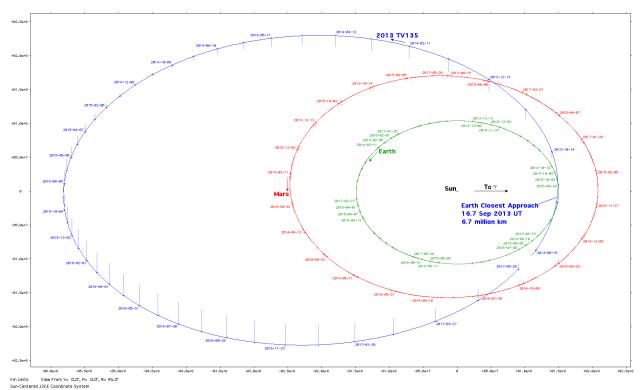


Figure 1. Motion of the Earth (green), Mars (red), and NEO 2013 TV_{135} (blue) is plotted relative to the Sun from August 2013 until May 2017. This heliocentric motion is viewed from a perspective 45° above Earth's orbit plane, the ecliptic. Time ticks ("+" markers) are at 30-day intervals and are labeled with the corresponding date in YYYY-MM-DD format at 60-day intervals. Dotted lines emanating from time ticks are projections onto the ecliptic plane. From these projections, note how the 2013 TV_{135} orbit's ascending node on the ecliptic closely coincides with crossing Earth's orbit inbound toward perihelion.

By the time 2013 TV_{135} was discovered, its Earth distance had increased to over 17 million km. To be observable from such a distance requires a NEO be relatively large, even if its reflectivity

^{*} Reference the NASA/JPL NEO Program Office's article on 2013 TV₁₃₅ and its Earth collision prospects at http://neo.jpl.nasa.gov/news/news180.html dated 17 October 2013.

[†] Unless noted otherwise, all 2013 TV₁₃₅-related data presented in this paper are obtained from JPL's *Horizons* ephemeris computation service. *Horizons* may be accessed at http://ssd.jpl.nasa.gov/?horizons.

is high. Current best estimates place 2013 TV₁₃₅'s diameter at about 400 m based on its absolute magnitude H = +19.4. When a NEO is brighter than (numerically less than) H = +22.0 and its minimum orbit intersection distance with Earth's orbit (MOID) is less than 7.5 million km, it is classified as a potentially hazardous object (PHO). With a MOID of 1.8 million km, 2013 TV₁₃₅ has become one of the 1435 PHOs known on 23 October 2013[‡].

As October 2013 draws to a close, very little of 2013 TV_{135} 's orbit has been observed. Consequently, future position predictions rapidly grow in uncertainty. Twenty years in the future, this PHO could plausibly occupy half of its orbit along an arc known as the line of variations (LOV)[§]. On 26 August 2032, Earth is very close to the ascending node where 2013 TV_{135} 's orbit crosses the ecliptic plane from below to above in Figure 1. Positions near the ascending node undergoing Earth collision are among the LOV points 2013 TV_{135} could occupy at that time.

As additional 2013 TV₁₃₅ observations are obtained, its LOV on 26 August 2032 will contract. At first, this will cause the probability of Earth collision P_C to increase. A maximum P_C of 1-in-10,000 (0.0001) or more will be reached as LOV points that do *not* collide are excluded. But the most likely long-term outcome is the tiny subset of LOV points that *do* collide with Earth will also be excluded, and P_C in 2032 will fall to zero.

The remainder of this paper develops geometric visualizations of how the 2013 TV_{135} LOV can collide with Earth on 26 August 2032 assuming recent knowledge of this PHO's orbit. Hopefully, these visualizations will only be of academic interest in the future.

2. Liminal Earth-Mapped Uncertainty Region (LEMUR) Analysis

Among the best available sources of P_C data on PHOs is the Sentry processor run by JPL for NASA's NEO Program Office^{**}. Both Sentry and similar Monte Carlo analyses randomly sample the LOV for potential Earth collision cases. The LEMUR analysis described here uses the JPL *Horizons* ephemeris computation service to systematically sample the LOV for collision cases previously identified by Sentry. These cases are coasted forward in time by *Horizons* as user-specified small bodies (USSBs) to a terminal epoch several hours before collision. A USSB coast is subject to the same JPL solar system standard dynamical modeling used by Sentry.

Geocentric position and velocity at the USSB terminal epoch are then coasted to Earth impact at a height of +42 km with the WeavEncke predictor^{††} modeling accelerations from Earth, Sun, and Moon gravity. The +42 km impact height is reckoned with respect to a *Horizons* spherical Earth radius of 6378.136 km and is consistent with the 6420 km marginal or grazing impact distance

[‡] Reference http://neo.jpl.nasa.gov/neo/groups.html for PHO criteria (synonymous with "PHA" on this web page) and a current tally of known PHOs.

[§] At a given instant, the LOV is a one-dimensional subset of all possible future positions. The LOV is restricted to lie on the nominal "best guess" orbit, but other plausible locations lie adjacent to it.

^{**} Sentry analysis is described at http://neo.jpl.nasa.gov/risk/doc/sentry.html.

^{††} Reference Adamo, D. R., "A Precision Orbit Predictor Optimized for Complex Trajectory Operations," *Astrodynamics 2003, Advances in the Astronautical Sciences*, Vol. 116, Univelt, San Diego, CA, 2003, pp. 2567–2586.

given in Sentry's "Impact Table Legend"^{‡‡}. Final conditions from each WeavEncke coast provide geodetic impact data for visualization purposes.

The LEMUR analysis documented here for 2013 TV₁₃₅ uses *Horizons* data in Table 1 to define the nominal orbit along which the LOV will be sampled. Seven Table 1 parameters appearing in **bold** are used to create USSB orbits in *Horizons* sampling the 2013 TV₁₃₅ LOV. These parameters are defined in Table 2.

Table 1. This 2013 TV_{135} orbit solution was obtained on 21.1 October 2013 UT and incorporates 180 observations spanning 12 days, as noted in its second line of data. The solution is tagged "JPL#7" in the second-to-last line of data. Parameters in bold are defined in Table 2.

```
*****
JPL/HORIZONS (2013 TV135) 2013-Oct-21 07:50:50
Rec #:752349 (+COV) Soln.date: 2013-Oct-21_03:15:08 # obs: 180 (12 days)
                                 (2013 TV135)
FK5/J2000.0 helio. ecliptic osc. elements (au, days, deg., period=Julian yrs):
  EPOCH= 2456584.5 ! 2013-Oct-19.00 (CT) Residual RMS= .34054
   EC= .5933521715952487 QR= .9954267524114992 TP= 2456555.7240458783
   OM= 333.4318585954334 W= 23.70784723315245 IN= 6.766820228742517
   A= 2.447884097442456 MA= 7.405383086758426 ADIST= 3.900341442473412

      PER= 3.82996
      N= .257346215
      ANGMOM= .021664166

      DAN= 1.02773
      DDN= 3.47271
      L= 356.9926052

      B= 2.7154528
      MOID= .0118506
      TP= 2013-Sep-20.2240458783

Asteroid physical parameters (km, seconds, rotational period in hours):

        GM= n.a.
        RAD= n.a.
        ROTPER= n.a.

        H= 19.418
        G= .150
        B-V= n.a.

                           G= .150
                           ALBEDO= n.a. STYP= n.a.
ASTEROID comments:
1: soln ref.= JPL#7, PHA OCC=7
2: source=ORB
*****
```

Table 2. Definitions are provided for Table 1 parameters appearing in bold. This dataset is used to create USSP orbits in *Horizons* sampling the 2013 TV_{135} LOV.

Parameter	Definition		
EPOCH	Coordinate Time (CT) ^{§§} expressed as a Julian date at which all other Table 2		
	parametric values are valid		
EC	Eccentricity		
QR TP	Perihelion distance in AU		
	CT of perihelion passage expressed as a Julian date		
OM	Ecliptic longitude of the ascending node on the ecliptic in deg		
W	Argument of the perihelion in deg		
IN	Ecliptic inclination in deg		

^{‡‡} This legend is displayed with PHO-specific Sentry data. The latest data for 2013 TV₁₃₅ can be found at http://neo.jpl.nasa.gov/risk/2013tv135.html.

^{§§} A uniform time scale void of leap seconds, CT is used as the fundamental ephemeris argument by *Horizons*. To a precision of ± 0.002 s, CT is related to international atomic time (TAI) by CT = TAI + 32.184 s.

Excepting EPOCH (a specified parameter), *Horizons* provides ± 1 sigma variance data for all Table 2 statistically estimated parameters associated with the latest orbit solution. In principle, the entire 2013 TV₁₃₅ uncertainty region can be sampled by introducing deviations from nominal values in Table 1 that are plausible with respect to their variances^{***}. Assuming no significant perturbations, such as would be incurred from a close planetary encounter, effects of these deviations over time fall into three classes. Deviations in EC or QR produce secular effects because either change implicitly alters nominal orbit period and can significantly depart from nominal position given sufficient coast time from EPOCH. In contrast, a TP deviation introduces a bias from nominal position along the LOV at EPOCH, and this bias does not grow significantly with time from EPOCH. Finally, deviations in OM, W, and IN produce bounded periodic departures from nominal position as a function of time from EPOCH.

Because the USSB coast time from EPOCH to Earth collision spans multiple 2013 TV₁₃₅ orbits, EC is selected for LOV sampling in this LEMUR analysis. Sentry analysis utilizing Table 1's JPL#7 orbit solution finds Earth collision cases at +1.28653 sigma from nominal position along the LOV. The ±1 sigma variance in EC from *Horizons* for JPL#7 is ±0.001070098. Thus, a good starting guess at an LOV sample colliding with Earth would be a USSB having Table 1 values with an EC deviation to 0.5933521715952487 - 1.28653 * 0.001070098 = 0.591975459^{†††}. Table 3 provides results from sampling the JPL#7 LOV with EC deviations.

Table 3. Earth miss distances from EC deviations sampling the 2013 TV₁₃₅ JPL#7 orbit solution's LOV are listed in order of increasing EC. The EC interval giving rise to all possible Earth impacts from this LEMUR analysis lies between the marginal leading and trailing graze cases. Values in the σ column are EC deviations from the nominal Table 1 value normalized to the *Horizons* variance of ±0.001070098.

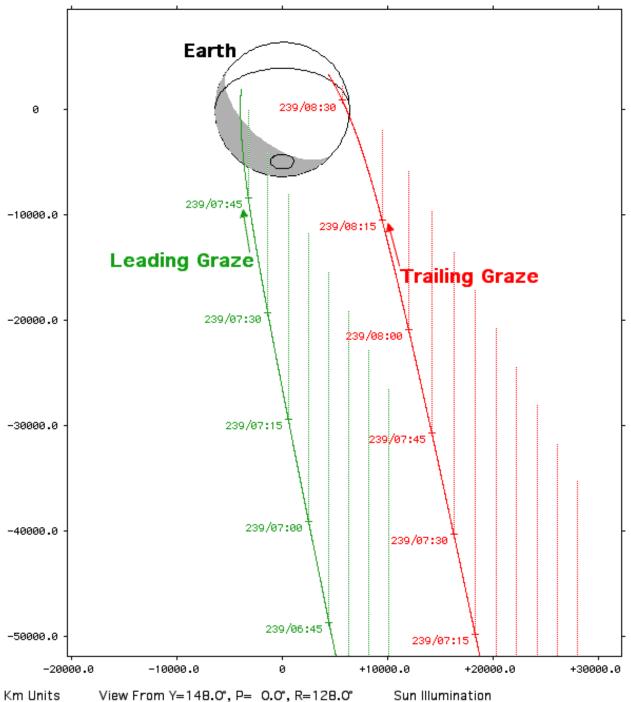
EC	σ	Earth Miss (km)	Remark
0.5919754590	-1.28653	679,354	Sentry-based guess
0.5919837773	-1.27876	6421	Leading graze
0.5919840948	-1.27846	6421	Trailing graze
0.5933521716	0	105,271,694	Nominal

In Table 3's LOV sampling, the Sentry-based guess for an EC deviation leading to Earth collision has a σ coordinate at -1.28653, only 0.6% from that of the nearest grazing case found by LEMUR analysis. Table 3 data also lead to a reasonable estimate of P_C . Assuming Gaussian probability density along the LOV, more than 99% of all possible positions are spanned from -3 sigma to +3 sigma. Consequently, $P_C \cong (1.27876 - 1.27846) / 6 = 0.000050$ or 1-in-20,000. The P_C computed by Sentry from JPL#7 data is 0.000054 or 1-in-18,000. In Figure 2, Table 3's two grazing cases are plotted relative to Earth. The leading graze impacts at 07:57 CT and the trailing graze at 08:34 CT. Sentry's predicted impact is at 08:24 CT.

^{***} Sentry samples deviations out to ± 5 sigma along the LOV.

^{†††} The sampled EC value for Earth collision is less than the nominal EC value because of Sentry's sign convention for displacement along the LOV with respect to nominal position. Positive Sentry LOV displacement signifies "displace from nominal in the direction of motion". To impart such a displacement requires a shorter orbit period than nominal. If QR (the only other Table 2 parameter capable of affecting orbit period) is held at its nominal value, EC must be decreased to shorten orbit period.

Potentially Hazardous Object (PHO) 2013 TV₁₃₅



Earth-Centered EPM Coordinate System @ 2032y 239d (8-26) 5:58:50 UTC

Figure 2. Grazing impact trajectories are plotted relative to Earth and delimit the minute 2013 TV_{135} LOV segment containing all possible collision cases from this LEMUR analysis. Time ticks are at 15-min intervals and are annotated with 26 August 2032 UT in DOY/hh:mm format. Dotted lines are projections onto Earth's equatorial plane. Earth is annotated with its equator and the parallel at 80° S latitude. The shaded area is Earth's nightside.

From JPL#7, Sentry also estimates 2013 TV₁₃₅ lateral position uncertainty from the LOV to 1 sigma confidence on 26 August 2032 as a "semi-width" of 0.0839 Earth radii or \pm 535 km. Bearing in mind this uncertainty, the LEMUR locus of all possible Earth collision points on 26 August 2032 is plotted in Figure 3.



Figure 3. The LEMUR locus of all possible Earth collisions for 2013 TV₁₃₅ on 26 August 2032 is confined to marine regions about southern Africa and adjacent Antarctic territory. This mapping is subject to 2013 TV₁₃₅ predicted position uncertainties and could be in error by 535 km or more in either lateral direction from the LEMUR locus.

3. Summary

In the weeks following PHO 2013 TV_{135} discovery, predicted position uncertainty admits a remote possibility of Earth collision on 26 August 2032. Systematic LEMUR analysis of this possible collision has been documented and shown to agree with NASA's automated Sentry software results. Although Sentry output is publicly accessible, geometric insights from LEMUR analysis are not generally available as of this writing. Chief among these insights is a LEMUR locus of possible impact points confined to marine regions about southern Africa and adjacent Antarctic territory.